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Kalkulus Peubah Banyak (C) Deadline : 27 Sept 2023

16. C. $9x^2 + 4y^2 - 36z = 0$, $3x + y + z - z^2 - 1 = 0$ di titik $(2, -3, 2)$

Dapatkan persamaan garis singgung dan bidang normal kurva!

misal : $F : 9x^2 + 4y^2 - 36z = 0$ titik $P_0\ (2, -3, 2)$

$G : 3x + y + z - z^2 - 1 = 0$

$\bullet\!> \dfrac{\partial F}{\partial x} = 18x$, $\bullet\!> \dfrac{\partial F}{\partial y} = 8y$ $\bullet\!> \dfrac{\partial F}{\partial z} = -36$

$\bullet\!> \dfrac{\partial G}{\partial x} = 3$ $\bullet\!> \dfrac{\partial G}{\partial y} = 1$ $\bullet\!> \dfrac{\partial G}{\partial z} = 1 - 2z$

▶ Persamaan Garis Singgung

$$\frac{x - x_0}{\begin{vmatrix} \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \end{vmatrix}_{P_0}} = \frac{y - y_0}{\begin{vmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial x} \\ \frac{\partial G}{\partial z} & \frac{\partial G}{\partial x} \end{vmatrix}_{P_0}} = \frac{z - z_0}{\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix}_{P_0}}$$

$$\frac{x - 2}{\begin{vmatrix} 8y & -36 \\ 1 & 1-2z \end{vmatrix}_{P_0}} = \frac{y + 3}{\begin{vmatrix} -36 & 18x \\ 1-2z & 3 \end{vmatrix}_{P_0}} = \frac{z - 2}{\begin{vmatrix} 18x & 8y \\ 3 & 1 \end{vmatrix}_{P_0}}$$

$$\frac{x - 2}{\begin{vmatrix} -24 & -36 \\ 1 & -3 \end{vmatrix}} = \frac{y + 3}{\begin{vmatrix} -36 & 36 \\ -3 & 3 \end{vmatrix}} = \frac{z - 2}{\begin{vmatrix} 36 & -24 \\ 3 & 1 \end{vmatrix}}$$

$$\frac{x-2}{108} = \frac{y+3}{216} = \frac{z-2}{108}$$

$\Rightarrow \bullet\!> \dfrac{x-2}{\cancel{108}} = \dfrac{z-2}{\cancel{108}}$ $\bullet\!> \dfrac{x-2}{\cancel{108}} = \dfrac{y+3}{\cancel{216}\ 2}$

$x - 2 = z - 2$ $2x - 4 = y + 3$

$x = z$ $y = 2x - 7$

Maka, persamaan Garis Singgungnya adalah perpotongan bidang $x = z$ dan $y = 2x - 7$

Persamaan Garis Singgung :

$$\begin{cases} x = z \\ y = 2x - 7 \end{cases}$$

▶ Bidang Normal

$$(x - x_0)\begin{vmatrix} \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \\ \frac{\partial G}{\partial y} & \frac{\partial G}{\partial z} \end{vmatrix} + (y - y_0)\begin{vmatrix} \frac{\partial F}{\partial z} & \frac{\partial F}{\partial x} \\ \frac{\partial G}{\partial z} & \frac{\partial G}{\partial x} \end{vmatrix} + (z - z_0)\begin{vmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \end{vmatrix} = 0$$

$$= (x-2)\begin{vmatrix} -24 & -36 \\ 1 & -3 \end{vmatrix} + (y+3)\begin{vmatrix} -36 & 36 \\ -3 & 3 \end{vmatrix} + (z-2)\begin{vmatrix} 36 & -24 \\ 3 & 1 \end{vmatrix} = 0$$

$$= (x-2)\,108 + (y+3)\,216 + (z-2)\,108 = 0$$

$$x - 2 + 2y + 6 + z - 2 = 0$$

$$x + 2y + z + 2 = 0$$

Jadi, persamaan bidang normal adalah $x + 2y + z + 2 = 0$

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17 a). Persamaan Bidang Singgung dan Garis Normal pada permukaan bidang lengkung $x^2 + y^2 + z^2 = 14$ di $(1, -2, 3)$

misal : $F : x^2 + y^2 + z^2 - 14 = 0$

$F_x|_{(1,-2,3)} = 2x = 2$

$F_y|_{(1,-2,3)} = 2y = 2(-2) = -4$

$F_z|_{(1,-2,3)} = 2z = 2(3) = 6$

▶ Persamaan Garis Normal

$$\frac{x - x_0}{F_x(x_0, y_0, z_0)} = \frac{y - y_0}{F_y(x_0, y_0, z_0)} = \frac{z - z_0}{F_z(x_0, y_0, z_0)}$$

$$\frac{x-1}{2} = \frac{y+2}{-4} = \frac{z-3}{6}$$

$\Rightarrow$ $\frac{x-1}{\cancel{2}} = \frac{z-3}{\cancel{6}\,3}$ $\qquad$ $\frac{x-1}{\cancel{2}} = \frac{y+2}{\cancel{-4}\,-2}$

$3x - 3 = z - 3$ $\qquad$ $-2x + 2 = y + 2$

$3x = z$ .... (1) $\qquad$ $y = -2x$

Persamaan Garis normal adalah perpotongan bidang $3x = z$ dan $y = -2x$

Jawab : $\begin{cases} 3x = z \\ y = -2x \end{cases}$ //

▶ Persamaan Bidang Singgung

$(x - x_0) F_x(x_0, y_0, z_0) + (y - y_0) F_y(x_0, y_0, z_0) + (z - z_0) F_z(x_0, y_0, z_0) = 0$

$(x-1).2 + (y+2)(-4) + (z-3).6 = 0$

$2x - 2 - 4y - 8 + 6z - 18 = 0$

$2x - 4y + 6z - 28 = 0$

Jadi, persamaan bidang Singgungnya adalah $2x - 4y + 6z - 28 = 0$ //

18. Ekspansikan fungsi berikut menurut deret Taylor / deret Maclaurin pada titik yang diberikan.

b). $f(x,y) = 2xy + x^2 + x^3$ di titik $(1,2)$

$\Rightarrow f_x(1,2) = 2y + 2x + 3x^2$
$= 4 + 2 + 3$
$= 9$

$\Rightarrow f_y(1,2) = 2x$
$= 2$

$\Rightarrow F_{xx}(1,2) = 2 + 6x$
$= 2 + 6$
$= 8$

$\Rightarrow F_{xy}(1,2) = 2$

$\Rightarrow F_{yy}(1,2) = 0$

$\Rightarrow F_{xxx}(1,2) = 6$

$\Rightarrow F_{xxy}(1,2) = 0$

$\Rightarrow F_{xyy} = 0$

$\Rightarrow F_{yyy} = 0$

▶ Deret Taylor $F(x,y)$ di sekitar (a,b)

$$F(1,2) = F(1,2) + \left[F_x(1,2)(x-1) + F_y(1,2)(y-2)\right] + \frac{1}{2}\left[F_{xx}(1,2)(x-1)^2 + 2F_{xy}(1,2)(x-1)(y-2)\right.$$
$$\left. + F_{yy}(1,2)(y-2)^2\right] + \frac{1}{3!}\left[F_{xxx}(1,2)(x-1)^3 + 3F_{xxy}(1,2)(x-1)^2(y-2)^2 + 3F_{xyy}(1,2)(x-1)(y-2)^2\right.$$
$$\left. + F_{yy}(1,2)(y-2)^3\right] + \ldots$$
$$= [4+1+1] + [9(x-1) + 2(y-2)] + \frac{1}{2}\left[8(x-1)^2 + 2.2(x-1)(y-2) + 2(y-2)^2\right]$$
$$\frac{1}{6}\left[6(x-1)^3 + 3(0) + 3(0) + 0\right] + \ldots$$
$$= 6 + [9(x-1) + 2(y-2)] + \frac{1}{2}\left[8(x-1)^2 + 4(x-1)(y-2) + 0\right] + \frac{1}{6}\left[(x-1)^3 + 0 + 0\right] + \ldots$$

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c). $f(x,y) = e^{x+y}\sin(x+y)$ di $(0,0)$

▶ Deret Mac laurin

$$F(x,y) = F(0,0) + \left[x F_x(0,0) + y F_y(0,0)\right] + \frac{1}{2}\left[x^2 F_{xx}(0,0) + 2xy F_{xy}(0,0) + y^2 F_{yy}(0,0)\right] + \ldots$$

- $f(0,0) = 0$
- $f_x(x,y) = e^x e^y \sin(x+y) + e^{x+y}\cos(x+y)$ $\quad f_x(0,0) = 1$
- $f_y(x,y) = e^{x+y}\sin(x+y) + e^{x+y}\cos(x+y)$ $\quad f_x(0,0) = 1$
- $f_{xx}(x,y) = 2e^{x+y}\cos(x+y)$ $\quad \Rightarrow f_{xx}(0,0) = 2$
- $f_{xy}(x,y) = 2e^{x+y}\cos(x+y)$ $\quad f_{xy}(0,0)\ 2$
- $f_{yy}(x,y) = 2e^{x+y}\cos(x+y)$ $\quad f_{yy}(0,0) = 2$

$$e^{x+y}\sin(x+y) = 0 + \left(x . f_x(0,0) + y f_y(0,0)\right) + 1/2\left[x^2 f_{xx}(0,0) + 2xy f_{xy}(0,0) + y^2 f_{yy}(0,0)\right]$$

Jadi menurut Deret Maclaurin :

$$e^{x+y}\sin(x+y) = 0 + [x+y] + 1/2\left[x^2 . 2 + 2xy . 2 + y^2 . 2\right] + \ldots$$
$$e^{x+y}\sin(x+y) = (x+y) + 1/2\left(2x^2 + 4xy + 2y^2\right) + \ldots + \ldots$$

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19. Kembangkan menurut deret Taylor

a. $f(x,y,z) = xz + y^2$ di sekitar $(1,-1,2)$ $\quad f(x,y,z) = xz + y^2 \longrightarrow f(1,-1,2) = 3$

- $f_x = z \longrightarrow f_x(1,-1,2) = 2$
- $f_y = 2y \longrightarrow f_y(1,-1,2) = -2$
- $f_z = x \longrightarrow f_z(1,-1,2) = 1$
- $f_{xy} = 0$
- $f_{xz}(x,y,z) = 1 \longrightarrow f_z(1,-1,2) = 1$
- $f_{yz}(x,y,z) = 0$
- $f_{xx}(x,y,z) = 2$
- $f_{yy}(x,y,z) = 2$
- $f_{zz}(x,y,z) = 0$

KIKY

Deret Taylor

$f(x,y,z) = f(1,-1,2) + [(x-1) f_x (1,-1,2) + (y+1) f_y (1,-1,2) + (z-2) f_z (1,-1,2)] +$
$\frac{1}{2} [(x-1)^2 f_{xx} (1,-1,2) + (y+1)^2 f_{yy} (1,-1,2) + (z-2) f_{zz} (1,-1,2) + 2$
$2(x-1)(y+1) f_{xy} (1,-1,2) + 2(x-1)(z-2) f_{xz} (1,-1,2) + 2(y+1)(z-2) f_{yz} (1,-1,2) + \ldots$
$= 3 + [(x-1).2 + (y+1).(-2) + (z-2).1 + \frac{1}{2} [(x-1)^2.0 + (y+1)^2.2 + (z-2).0 +$
$2(x+1)(y+1).0 + 2(x-1)(z-2).1 + 2(y+1)(z-2).0] + \ldots$
$xz + y^2 = 3 + [2(x-1) + (-2)(y+1) + (z-2)] + \frac{1}{2!} [2(y+1)^2 + 2(x-1)(z-2)] + \ldots$

20. Gunakan Deret Taylor sampai dengan suku derajat dua saja untuk mendapatkan nilai

a. $\sqrt{1{,}03} \cdot \sqrt[3]{0{,}98}$

→ Deret Taylor derajat dua

$$f(x) = f(a) + f_x(a)(x-a) + \frac{1}{2!} f_{xx}(a)(x-a)^2$$

dengan $a = 1$, Karena a pendekatan $\sqrt{1{,}03}$ dan $\sqrt[3]{0{,}98}$

- Deret Taylor pada $\sqrt{1{,}03}$ dengan $x = 1{,}03$

$f(x) = \sqrt{x}$ $\qquad f_{xx}(x) = -\frac{1}{4} x^{-3/2}$

$f(a) = \sqrt{1} = 1$ $\qquad f_{xx}(a) = -\frac{1}{4} \cdot 1^{-3/2} = -\frac{1}{4}$

$f_x(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$f_x(x) = \frac{1}{2}$

Jadi, $\sqrt{1{,}03} \approx 1 + \frac{1}{2}(0{,}3) + \frac{1}{2}(-1/4)[1{,}03-1]^2$

$\approx 1 + \frac{3}{20} - \frac{1}{8}\left[\frac{9}{10000}\right]$

$\approx \frac{81191}{80.000} \approx 1{,}01489$

- Deret Taylor pada $\sqrt[3]{0{,}98}$ dengan $x = 0{,}98$

$f(x) = \sqrt[3]{x}$ $\qquad f_{(x,x)}(x) = -\frac{2}{9} \cdot x^{-5/3}$

$f_x(x) = \frac{1}{3} \cdot \frac{1}{\sqrt[3]{x^2}}$ $\qquad f_{(x,x)}(a) = -\frac{2}{9} 1^{-5/3} = -\frac{2}{9}$

$f_x(1) = \frac{1}{3}$. $\qquad f(a) = \sqrt[3]{1} = 1$

Jadi, $\sqrt[3]{0{,}98} \approx 1 + 1/3\,(0{,}98-1) + 1/2\,(-2/9)(0{,}98-1)^2$

$\approx 1 - \frac{1}{50} - \frac{1}{9}(2500)]$

$\approx \frac{22344}{22500} \approx 0{,}993289$

Jadi, nilai aproksimasi $\sqrt{1{,}03} \cdot \sqrt[3]{0{,}98} \approx 1{,}01489 \cdot 0{,}993289$

$\approx 1{,}008079073 2 4$

$\approx 1{,}0081$ //

KIKY

b. $0,95^{2,01}$

1▶ Deret Taylor derajat Dua

$f(x) = f(a) + f_x(a)\,(x-a) + \frac{1}{2!} f_{xx}(a)\,(x-a)^2$

dengan $a = 1$, karena kita hendak mencari aproksimasi nilai $0,95^{2,01}$

$f(x) = x^{2,01}$

$f(a) = 1^{2,01} = 1$

$f_x(x) = 2,01 \cdot x^{1,01}$

$f_x(a) = 2,01 \cdot 1^{1,01}$

$= 2,01$

$f_{xx}(x) = 2,0\ \cdot x^{0,01}$

$= 2,0301 \cdot x^{0,01}$

$f_{xx}(a) = 2,0301 \cdot 1^{0,01}$

$= 2,0301$

Jadi, nilai dari $0,95^{2,01} \approx 1 + 2,01\,(0,95-1) + \frac{1}{2!}\,2,0301\,(0,95-1)^2$

$\approx 1 - 0,1005 + 0,0025\,37625$

$\approx 1 - 0,09706\,2375$

$\approx 0,902$ //